TRUSS STRUCTURE

(1) Parameters

	Content
NNODE	The maximum value of the number of node
NUMB	The maximum value of the number of member
NTDIS	The maximum value of the number of degree of freedom
NDIM	2 The number of dimension (plan truss element = 2)
NFREE	2 The number of the degrees of freedom per one node
MFREE	4 The number of the degrees of freedom per one element (MFREE = NFREE*NCON)
NCON	2 The number of the nodes per one member

(2) Variables

	Content
NODE	The number of node (NODE≦NNODE)
MEMB	The number of member (MEMB≦NUMB)
NSK	The number of degree of freedom (NSK≦NTDIS)

(3) Dimension [main program]

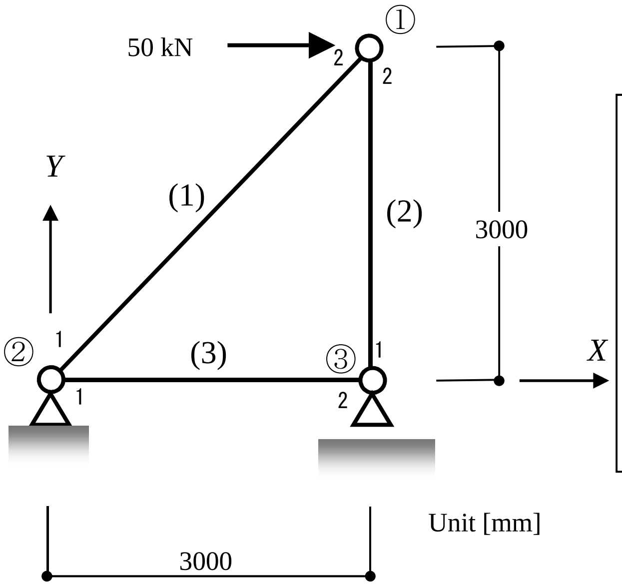
	Content
RA(NUMB)	Cross-sectional area of a member
RL(NUMB)	Member length of a member
YOUNG(NUMB)	Young's modulus of a member
TZ(NUMB)	The member rotation angle of a member θ
COSTZ(NUMB)	The value of cos θ
SINTZ(NUMB)	The value of sin θ
COOD(NDIM,NNODE)	Nodal coordinates COOD (1, M) : X coordinate of Node M COOD (2, M) : Y coordinate of Node M
PNODE(NFREE,NNODE)	Nodal applied load PNODE(1,M) : Applied load which acts in the direction of X of Node M PNODE(2,M) : Applied load which acts in the direction of Y of Node M
PLOAD(NTDIS)	Load vector
SMJ(NTDIS,NTDIS)	Overall stiffness matrix
SM(MFREE,MFREE)	Stiffness matrix of a member
ROT(MFREE,MFREE)	Rotation (transfer) matrix of a member
DISP(NTDIS)	Displacement vector
IRE(NFREE,NNODE)	Table of node restricted condition (= 0 : fix, = 1 : free)
IRO(NFREE,NNODE)	Table of unknown nodal displacement of node (= 0 : fix, = 1 : free)
INO(MFREE,NUMB)	Table of unknown nodal displacement of member (= 0 : fix, = 1 : free)
ICON(NCON,NUMB)	Table of member combination ICON(1,M) : The node number of *i* end of member M ICON(2,M) : The node number of *j* end of member M

Input manual

1. Component of Input Data

The input data of this program is carried in order of the following; (1) The input about nodal coordinates and restricted conditions, (2) The input about members and (3) The input about loading conditions. The method of making of input data is explained by making into an example the truss structure (exercise 1) shown in Fig.1.

Exercise 1



The number of node : 3,
The number of member : 3

Member (1)
Young's modulus: E = 205 kN/mm^2
Area: A = 2000 mm^2

Member (2), (3) :
Young's modulus: E = 205 kN/mm^2
Area: A = 1000mm^2

Figure 1 Truss structure

2. Input of Nodal Coordinates and Restricted Conditions

In Subroutine INNODE, the nodal coordinates and the restricted conditions of the structure are inputted.

2.1 Input of Nodal Coordinates

First, the number of nodes of the structure is inputted.

Column	Format	Input data	Comment
*	*	NODE	The number of nodes of the structure（NODE≦NNODE）

Next, only the number of nodes NODE, the input of the nodal coordinates is repeated.

Column	Format	Input data	Comment
*	*	J	Node number
*	*	COOD(J, 1)	X coordinates of Node J
*	*	COOD(J, 2)	Y coordinates of Node J

2.2 Input of Restricted Conditions of Nodes

The number NRZERO of nodes which should input restricted conditions is inputted.

Column	Format	Input data	Comment
*	*	NRZERO	The number of nodes which should input restricted conditions

Next, only the number of nodes NRZERO, the input of the restricted condition index, IRE, of node is repeated.

Column	Format	Input data	Comment
*	*	J	Node number
*	*	IRE(1, J)	Restricted condition index of X direction of Node J
*	*	IRE(2, J)	Restricted condition index of Y direction of Node J

IRE(*, *)	= 1	Free
	= 0	Fix
	= −10×N + L	Same displacement as the displacement of L direction of Node N

The input example of nodal coordinates and restricted conditions is shown below.

input example : nodal coordinates and restricted conditions		
3		
1	3000.0	3000.0
2	0.0	0.0
3	3000.0	0.0
3		
1	1	1
2	0	0
3	1	0

3. Input of member properties and conditions of member connection

In Subroutine INMEMB, the cross-sectional properties of members and conditions of member connection are inputted.

First, the number MEMB of members used for analysis is inputted.

Column	Format	Input data	Comment
*	*	MEMB	The number of members （≦ NUMB ）

Next, only the number of members MEMB, the connection information on each member, the Young's modulus and the cross-section area of each member are inputted.

Column	Format	Input data	Comment
*	*	J	Member number
*	*	ICON(J, 1)	The node number of the 1st end of Member J
*	*	ICON(J, 2)	The node number of the 2nd end of Member J
*	*	YOUNG(J)	Young's modulus of Member J
*	*	RA(J)	Cross-section area of Member J

The input example of member properties and conditions of member connection is shown below.

input example: member properties and conditions of member connection				
3				
1	2	1	205. 0	2000. 0
2	3	1	205. 0	1000. 0
3	2	3	205. 0	1000. 0

4. Input of load conditions

In Subroutine INLOAD, the conditions of the applied loads are inputted.

First, the number N of nodes on which nodal force acts is inputted.

Column	Format	Input data	Comment
*	*	N	The number of nodes on which nodal force acts

Next, only the number of nodes N, the input of applied forces is repeated.

Column	Format	Input data	Comment
*	*	J	Node number
*	*	PNODE(1, J)	Applied load of X direction of Node J
*	*	PNODE(2, J)	Applied load of Y direction of Node J

The input example of applied load conditions is described below.

input example: applied load conditions		
1		
1	50. 0000	0. 0000

5. Output

The output result of this program is outputted to standard output (screen output and equipment number =6) and the "OUTPUT" file. "OUTPUT OF NODAL DISPLACEMENT" expresses nodal displacement, the row of "DISP(X)" and “DISP(Y) express the displacements of X and Y direction, respectively.

Example of「OUTPUT」

```
**** OUTPUT OF NODAL DISPLACEMENT ****
NODE =          3
---------------------------------
 NODE      DISP(X)      DISP(Y)
    1  0.1766E+01 -0.7317E+00
    2  0.0000E+00  0.0000E+00
    3  0.0000E+00  0.0000E+00
---------------------------------


**** OUTPUT OF AXIAL FORCE ****
MEMB =          3
---------------------------------------------------
MEMBER   AXIAL FORCE         STRESS         STRAIN
    1      0.7071E+02     0.3536E-01     0.1725E-03
    2     -0.5000E+02    -0.5000E-01    -0.2439E-03
    3      0.0000E+00     0.0000E+00     0.0000E+00
---------------------------------------------------
```

Example of output of display

```
NODE =     3
**** COOD ( SUB.INNODE ) ****
         1  3000.000  3000.000
         2     0.000     0.000
         3  3000.000     0.000

NRZERO =     3
**** SUB.INNODE : IRE ( ) ****
         1         1         1
         2         0         0
         3         1         0

MEMB =     3
**** OUTPUT OF SUB.INMEMB  ****
MEMB =         3
    1    2    1  0.2050E+03  0.2000E+04
    2    3    1  0.2050E+03  0.1000E+04
    3    2    3  0.2050E+03  0.1000E+04

**** SUB.INLOAD ****
    1  0.5000E+02  0.0000E+00
    2  0.0000E+00  0.0000E+00
    3  0.0000E+00  0.0000E+00

**** SUB.RNMRG : IRO( )****
NSK =         3
         1         1         2
         2         0         0
         3         3         0

**** SUB.RNMRGP : INO( ) ****
         1         0         0         1         2
         2         3         0         1         2
         3         0         0         3         0

**** SUB.LENGTH ****
MEMBER  LENGTH        SIN       COS
    1 4242.6406    0.7071    0.7071
    2 3000.0000    1.0000    0.0000
    3 3000.0000    0.0000    1.0000

*** SUB.STFEL *** [ K (     1 ) ]
  0.9664E+02  0.0000E+00 -0.9664E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
 -0.9664E+02  0.0000E+00  0.9664E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00

*** SUB.MKROT *** [ ROT (     1 ) ]
  0.7071E+00  0.7071E+00  0.0000E+00  0.0000E+00
 -0.7071E+00  0.7071E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.0000E+00  0.7071E+00  0.7071E+00
  0.0000E+00  0.0000E+00 -0.7071E+00  0.7071E+00
```

```
*** SUB.ADDSM *** [ K (     1 ) ]
INO =     0     0     1     2
  0.4832E+02  0.4832E+02 -0.4832E+02 -0.4832E+02
  0.4832E+02  0.4832E+02 -0.4832E+02 -0.4832E+02
 -0.4832E+02 -0.4832E+02  0.4832E+02  0.4832E+02
 -0.4832E+02 -0.4832E+02  0.4832E+02  0.4832E+02

*** SUB.STFEL *** [ K (     2 ) ]
  0.6833E+02  0.0000E+00 -0.6833E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
 -0.6833E+02  0.0000E+00  0.6833E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00

*** SUB.MKROT *** [ ROT (     2 ) ]
  0.0000E+00  0.1000E+01  0.0000E+00  0.0000E+00
 -0.1000E+01  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.1000E+01
  0.0000E+00  0.0000E+00 -0.1000E+01  0.0000E+00

*** SUB.ADDSM *** [ K (     2 ) ]
INO =     3     0     1     2
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.6833E+02  0.0000E+00 -0.6833E+02
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00 -0.6833E+02  0.0000E+00  0.6833E+02

*** SUB.STFEL *** [ K (     3 ) ]
  0.6833E+02  0.0000E+00 -0.6833E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
 -0.6833E+02  0.0000E+00  0.6833E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00

*** SUB.MKROT *** [ ROT (     3 ) ]
  0.1000E+01  0.0000E+00  0.0000E+00  0.0000E+00
  0.0000E+00  0.1000E+01  0.0000E+00  0.0000E+00
  0.0000E+00  0.0000E+00  0.1000E+01  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.1000E+01

*** SUB.ADDSM *** [ K (     3 ) ]
INO =     0     0     3     0
  0.6833E+02  0.0000E+00 -0.6833E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00
 -0.6833E+02  0.0000E+00  0.6833E+02  0.0000E+00
  0.0000E+00  0.0000E+00  0.0000E+00  0.0000E+00

*** SUB.STFGL ***
*** STIFNESS MATRIX NSK =          3 ***
  0.4832E+02  0.4832E+02  0.0000E+00
  0.4832E+02  0.1167E+03  0.0000E+00
  0.0000E+00  0.0000E+00  0.6833E+02

**** SUB.CALOAD ****
```

```
PLOAD(    1 ) =  0.5000E+02
PLOAD(    2 ) =  0.0000E+00
PLOAD(    3 ) =  0.0000E+00

*** SUB.OUTDISP : DISPLACMENT VECTOR ***
DISP(    1 ) =  0.1766E+01
DISP(    2 ) = -0.7317E+00
DISP(    3 ) =  0.0000E+00

*** DISPLACEMNT AND FORCE OF MEMBER =    1
    J   DISP(J)     R*DISP(J)    FORCE(J)
    1  0.0000E+00  0.0000E+00 -0.7071E+02
    2  0.0000E+00  0.0000E+00  0.0000E+00
    3  0.1766E+01  0.7317E+00  0.7071E+02
    4 -0.7317E+00 -0.1766E+01  0.0000E+00

MEMBER =    1
AXIAL FORCE N =  0.7071E+02
STRESS =  0.3536E-01
STRAIN =  0.1725E-03

*** DISPLACEMNT AND FORCE OF MEMBER =    2
    J   DISP(J)     R*DISP(J)    FORCE(J)
    1  0.0000E+00  0.0000E+00  0.5000E+02
    2  0.0000E+00  0.0000E+00  0.0000E+00
    3  0.1766E+01 -0.7317E+00 -0.5000E+02
    4 -0.7317E+00 -0.1766E+01  0.0000E+00

MEMBER =    2
AXIAL FORCE N = -0.5000E+02
STRESS = -0.5000E-01
STRAIN = -0.2439E-03

*** DISPLACEMNT AND FORCE OF MEMBER =    3
    J   DISP(J)     R*DISP(J)    FORCE(J)
    1  0.0000E+00  0.0000E+00  0.0000E+00
    2  0.0000E+00  0.0000E+00  0.0000E+00
    3  0.0000E+00  0.0000E+00  0.0000E+00
    4  0.0000E+00  0.0000E+00  0.0000E+00

MEMBER =    3
AXIAL FORCE N =  0.0000E+00
STRESS =  0.0000E+00
STRAIN =  0.0000E+00

**********************************************
* FINISH NO ERROR !!                         *
* 2D TRUSS PROGRAM  VERSION 6.0 (2016/01/30) *
**********************************************
```

Appendix: Comparison with a theoretical solution

【１】Calculation of the stiffness matrix of a member（STFGL）

(1) member(1) 【Node②→Node①】

$\ell_1 = 3000 \times \sqrt{2} = 4242.6406$ [mm]

$\theta_1 = 45$deg., $\cos\theta_1 = 1/\sqrt{2}$, $\sin\theta_1 = 1/\sqrt{2}$, $\cos^2\theta_1 = \frac{1}{2}$, $\sin^2\theta_1 = \frac{1}{2}$, $\sin\theta_1 \cos\theta_1 = \frac{1}{2}$

$$\frac{E_1 \cdot A_1}{\ell_1} = \frac{205 \times 2000}{4242.64} = 96.638 \text{ [kN/mm]}$$

Stiffness matrix of member (1) $[k_1]$

$$[k_1] = \frac{E_1 \cdot A_1}{\ell_1}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 96.638 & 0 & -96.638 & 0 \\ 0 & 0 & 0 & 0 \\ -96.638 & 0 & 96.638 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rotation matrix of member (1) $[R_1]$

$$[R_1] = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} = \begin{bmatrix} 0.7071 & 0.7071 & 0 & 0 \\ -0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \\ 0 & 0 & -0.7071 & 0.7071 \end{bmatrix}$$

Stiffness matrix $[K_1] = [R_1]^T [k_1] [R_1]$

$$[K_1] = \frac{E_1 \cdot A_1}{\ell_1}\begin{bmatrix} \cos^2\theta_1 & \sin\theta_1\cos\theta_1 & -\cos^2\theta_1 & -\sin\theta_1\cos\theta_1 \\ \sin\theta_1\cos\theta_1 & \sin^2\theta_1 & -\sin\theta_1\cos\theta_1 & -\sin^2\theta_1 \\ -\cos^2\theta_1 & -\sin\theta_1\cos\theta_1 & \cos^2\theta_1 & \sin\theta_1\cos\theta_1 \\ -\sin\theta_1\cos\theta_1 & -\sin^2\theta_1 & \sin\theta_1\cos\theta_1 & \sin^2\theta_1 \end{bmatrix}$$

$$= [K_1] = \begin{array}{c} \begin{bmatrix} 48.319 & 48.319 & -48.319 & -48.319 \\ 48.319 & 48.319 & -48.319 & -48.319 \\ -48.319 & -48.319 & 48.319 & 48.319 \\ -48.319 & -48.319 & 48.319 & 48.319 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix} \\ \begin{matrix} 0 & \quad 0 & \quad 1 & \quad 2 \end{matrix} \end{array}$$

(2) member(2) 【Node③→Node①】

$\ell_2 = 3000$ [mm] $\theta_2 = 90$deg., $\cos\theta_2 = 0$, $\sin\theta_2 = 1$, $\cos^2\theta_2 = 0$, $\sin^2\theta_2 = 1$, $\sin\theta_2 \cos\theta_2 = 0$

$$\frac{E_2 \cdot A_2}{\ell_2} = \frac{205 \times 1000}{3000} = 68.333 \text{ [kN/mm]}$$

Stiffness matrix $[k_2]$

$$[k_2] = \frac{E_2 \cdot A_2}{\ell_2}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rotation matrix $[R_2]$

$$[R_1] = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & 0 & \sin\theta_2 & \cos\theta_2 \end{bmatrix} = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{bmatrix}$$

Stiffness matrix $[K_2] = [R_2]^T [k_2] [R_2]$

$$[K_2] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 68.333 & 0 & -68.333 \\ 0 & 0 & 0 & 0 \\ 0 & -68.333 & 0 & 68.333 \end{bmatrix} \begin{matrix} 3 \\ 0 \\ 1 \\ 2 \end{matrix}$$
$$\begin{matrix} 3 & 0 & 1 & 2 \end{matrix}$$

(3) member(3)　【Node②→Node③】

ℓ_3 = 3000 [mm], θ_3 = 0deg.,　$\cos\theta_3 = 1$,　$\sin\theta_3 = 0$,　$\cos^2\theta_3 = 1$,　$\sin^2\theta_3 = 0$,　$\sin\theta_3\cos\theta_3 = 0$

$$\frac{E_3 \cdot A_3}{\ell_3} = \frac{205 \times 1000}{3000} = 68.333 \text{ [kN/mm]}$$

Stiffness matrix $[k_3]$

$$[k_3] = \frac{E_3 \cdot A_3}{\ell_3} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rotation matrix $[R_3]$

$$[R_3] = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & \cos\theta_3 & \sin\theta_3 \\ 0 & 0 & \sin\theta_3 & \cos\theta_3 \end{bmatrix} = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

Stiffness matrix $[K_3] = [R_3]^T [k_3][R_3]$

$$[K_3] = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 3 \\ 0 \end{matrix}$$
$$\begin{matrix} 0 & 0 & 3 & 0 \end{matrix}$$

【2】Calculation of the stiffness matrix of the whole structure（ADDSM）

$$[K] = \begin{bmatrix} 48.319 & 48.319 & 0 \\ 48.319 & 48.319 + 68.333 & 0 \\ 0 & 0 & 0 + 68.333 \end{bmatrix} = \begin{bmatrix} 48.319 & 48.319 & 0 \\ 48.319 & 116.652 & 0 \\ 0 & 0 & 68.333 \end{bmatrix}$$

【3】Calcuration of applined load vector（CALOAD）

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 50.00 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

【4】Solution of simultaneous equations（GAUSS）

$$\begin{bmatrix} 48.319 & 48.319 & 0 \\ 48.319 & 116.652 & 0 \\ 0 & 0 & 68.333 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{Bmatrix} 50.00 \\ 0.0 \\ 0.0 \end{Bmatrix}$$

$$\begin{Bmatrix} D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} 0.03533 & -0.01463 & 0 \\ -0.01463 & 116.652 & 0 \\ 0 & 0 & 0.014634 \end{bmatrix} \begin{Bmatrix} 50.00 \\ 0.0 \\ 0.0 \end{Bmatrix} = \begin{Bmatrix} 1.7665 \\ -0.7317 \\ 0.0 \end{Bmatrix}$$

【5】Output of nodal displacemnets（OUTDISP）

nodal displacemnets [mm]

Node No.	X direction	Y direction
1	1.7665	-0.7317
2	0.00	0.00
3	0.00	0.00

【6】Calcuration of axial forces（OUTSTR）

(1) member (1) 【Node②→Node①】

$$[k_1] = \begin{bmatrix} 96.638 & 0 & -96.638 & 0 \\ 0 & 0 & 0 & 0 \\ -96.638 & 0 & 96.638 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad , \quad [R_1] = \begin{bmatrix} 0.7071 & 0.7071 & 0 & 0 \\ -0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \\ 0 & 0 & -0.7071 & 0.7071 \end{bmatrix}$$

Nodal displacement vector of member (1) $\{D_1\}$

$$\{D_1\} = \begin{Bmatrix} 0 \\ 0 \\ D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ 1.7665 \\ -0.7317 \end{Bmatrix}$$

Nodal displacement vector of member (1) $\{d_1\}$

$$\{d_1\} = [R_1]\{D_1\} = \begin{bmatrix} 0.7071 & 0.7071 & 0 & 0 \\ -0.7071 & 0.7071 & 0 & 0 \\ 0 & 0 & 0.7071 & 0.7071 \\ 0 & 0 & -0.7071 & 0.7071 \end{bmatrix} \begin{Bmatrix} 0.00 \\ 0.00 \\ 1.7665 \\ -0.7317 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.73171 \\ -1.7665 \end{Bmatrix}$$

Nodal force vector of member (1) $\{f_1\}$

$$\{f_1\} = [k_1]\{d_1\} = \begin{bmatrix} 96.638 & 0 & -96.638 & 0 \\ 0 & 0 & 0 & 0 \\ -96.638 & 0 & 96.638 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.73171 \\ -1.7665 \end{Bmatrix} = \begin{Bmatrix} -70.7113 \\ 0.00 \\ 70.7113 \\ 0.00 \end{Bmatrix}$$

Axial force of member (1) N_1

$$N_1 = 70.711 \ [\text{kN}]$$

$$\sigma_1 = \frac{N_1}{A_1} = \frac{70.7113}{2000} = 0.03536 \ [\text{kN/mm}^2] \quad , \quad \varepsilon_1 = \frac{\sigma_1}{E_1} = \frac{0.035356}{205} = 0.0001725$$

(2) member(2) 【Node③→Node①】

$$[k_2] = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad , \quad [R_2] = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{bmatrix}$$

Nodal displacement vector of member (2) $\{D_2\}$

$$\{D_2\} = \begin{Bmatrix} D_3 \\ 0 \\ D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ 1.7665 \\ -0.7317 \end{Bmatrix}$$

Nodal displacement vector of member (2) $\{d_2\}$

$$\{d_2\} = [R_2]\{D_2\} = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ -1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.0 \\ 0 & 0 & -1.0 & 0 \end{bmatrix} \begin{Bmatrix} 0.00 \\ 0.00 \\ 1.7665 \\ -0.7317 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ -0.7317 \\ -1.7665 \end{Bmatrix}$$

Nodal force vector of member (2) $\{f_2\}$

$$\{f_2\} = [k_2]\{d_2\} = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0.00 \\ 0.00 \\ -0.7317 \\ -1.7665 \end{Bmatrix} = \begin{Bmatrix} 50.000 \\ 0.00 \\ -50.000 \\ 0.00 \end{Bmatrix}$$

Axial force of member (2) N_2

$$N_2 = 70.711 \ [\text{kN}]$$

$$\sigma_2 = \frac{N_2}{A_2} = \frac{50.000}{1000} = 0.05 \ [\text{kN/mm}^2] \quad , \quad \varepsilon_2 = \frac{\sigma_2}{E_2} = \frac{0.05}{205} = 0.00024$$

(3) member(3)　【Node②→Node③】

$$[k_3] = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad , \quad [R_3] = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

Nodal displacement vector of member (3)$\{D_3\}$

$$\{D_3\} = \begin{Bmatrix} 0 \\ 0 \\ D_3 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{Bmatrix}$$

Nodal displacement vector of member (3) $\{d_3\}$

$$\{d_3\} = [R_3]\{D_3\} = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix} \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{Bmatrix}$$

Nodal force vector of member (3) $\{f_3\}$

$$\{f_3\} = [k_3]\{d_3\} = \begin{bmatrix} 68.333 & 0 & -68.333 & 0 \\ 0 & 0 & 0 & 0 \\ -68.333 & 0 & 68.333 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{Bmatrix} = \begin{Bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{Bmatrix}$$

Axial force of member (3) N_3

$$N_3 = 0.000 \ [\text{kN}]$$

$$\sigma_3 = \frac{N_3}{A_3} = 0.00 \ [\text{kN/mm}^2] \quad , \quad \varepsilon_3 = \frac{\sigma_3}{E_3} = 0.00$$